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Oscillatory Marangoni convection in variable-viscosity fluid layer: The effect of thermal feedback control

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ABSTRACT

The effect of a feedback control strategy on the onset of oscillatory convection in an infinite horizontal layer of fluid with temperature-dependent viscosity is investigated theoretically using linear stability analysis. It is shown that small viscosity variations stabilizes the fluid layer. Large controller gains, large viscosity variations, and high surface tension, however, promote the onset of overstability leading to oscillatory motions.

1. Introduction

The time-dependent oscillatory state of Marangoni convection is known to be the primary cause of detrimental striations in the chemical composition of the final products in several material processing technologies such as the semiconductor crystal growth production. The role of thermocapillary (surface tension driven) flow in the heat and mass transfer processes, the characteristics of the transition from steady to oscillatory flow and effective mechanisms to reduce or alter the oscillatory flow are of fundamental and industrial interest. The pioneering works of Bénard [1], Rayleigh [2] and Pearson [3] have been extended to investigate comprehensively the behavior of the steady and oscillatory instabilities driven by buoyancy (Bénard) and thermocapillary (Marangoni) effects. Typically, in reduced gravity or in a sufficiently thin layer of fluid, the thermocapillary forces are dominant [4].

The problems of the onset of oscillatory convection in the presence of external forces such as magnetic field and rotation have been studied by many authors [5–10]. The effects of linear and nonlinear control strategies on the steady and oscillatory stability thresholds have been studied both experimentally and theoretically [13–19]. Bau [13] applied a linear control feedback to delay the onset of convection and investigate the possibility of a bifurcation through imaginary growth rate into oscillatory convection in the case of flat free surface. For a liquid layer heated from below with uniform temperature at the rigid wall, the oscillatory convection cannot appear [13,20–22], but in [13] large controller gains may induce oscillatory instabilities and have a destabilizing effect. Experiments by Tang and Bau [16] revealed that at relatively large controller gain, the controller itself introduced oscillatory behavior which then increased the amplitudes of the oscillations. Or et al. [17] employed a nonlinear feedback control strategy to delay the onset and eliminate the subcritical long-wavelength instability of Marangoni–Bénard convection. Or and Kelly [18] showed that the weakly nonlinear flow properties in the Rayleigh–Bénard–Marangoni problem can be altered by linear and nonlinear proportional feedback control processes and the stabilization of the basic state can be achieved. Remillieux et al. [19] delineated the mechanisms that lead to oscillatory Rayleigh–Bénard convection in the presence of large controller gains and the application of derivative controller to suppress oscillatory convection. Very recently, the effects of the combined rotation and feedback controller on the onset of steady and oscillatory Marangoni instability have been investigated in [10–12].

The dynamic viscosity in most fluids is generally sensitive to temperature variations which can also influence heat transport and the spatial structure of the fluids [23]. Furthermore, variable-viscosity fluids are less stable than a constant-viscosity fluid. The destabilizing effect of linear viscosity variation has been investigated in [24,25] for steady and oscillatory instabilities. However, liquids such as silicone oils and glycerol have strong viscosity variations which are usually well described by exponential laws and significantly alter the convective instabilities [22,26–32]. Slavtchev et al. [22] studied the influences of viscosity variation and deformable free surface on oscillatory instabilities, and found that in the case of a constant heat flux from below, oscillatory instability

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Nomenclature

d	initial thickness of the liquid layer
Bi	Biot number
Bo	Bond number
Cr	Crispation number
D	differentiation with respect to z
$f(z)$	function related to viscosity variation
g	gravitational acceleration
h	heat transfer coefficient
K	controller gain
Ma	Marangoni number
N	viscosity parameter
Pr	Prandtl number
$T(z)$	temperature
$W(z)$	vertical variation of velocity perturbation
z	vertical coordinate
Z	magnitude of free surface deflection

Greek symbols

α	total wave number
β, ϵ, γ	constants
χ	thermal diffusivity
λ	thermal conductivity
μ	dynamic viscosity
ν	kinematic viscosity
ω	time growth rate
ρ	density of the fluid
σ	surface tension
$\Theta(z)$	vertical variation of temperature perturbation

is expected to appear in medium viscous fluids having low surface tension. Awang Kechil and Hashim [33] applied the feedback control to overcome the destabilizing effect of viscosity variation and delay the onset of steady Marangoni convection.

In this paper, we extend the work of Slavtchev et al. [22] to include the effect of feedback control. We use classical linear stability analysis and obtain analytical solution for the Marangoni problem in a variable-viscosity fluid layer subject to a constant temperature at the lower boundary in the presence of a thermal feedback control. We investigate the possibility of suppressing or promoting oscillatory instabilities through the thermal feedback controller. The roles of physical parameters on the onset of oscillatory instability are also assessed.

2. Mathematical formulation

Consider the convective flow in a horizontal layer of incompressible viscous, heat-conducting fluid on a rigid plate with free upper surface. The surface tension σ and the dynamic viscosity μ are assumed to vary linearly and exponentially, respectively, with temperature,

$$\sigma = \sigma_0 - \epsilon(T - T_0), \tag{1}$$

$$\mu = \mu_0 \exp\left[-\gamma(T - T_0)\right], \tag{2}$$

where T is the fluid temperature, σ_0 and μ_0 are values at a reference temperature T_0, and γ and ϵ are positive constants. All other physical properties of the fluid are assumed constant. The bottom boundary is subjected to a no-slip condition and a uniform temperature or the so-called "conducting" case.

The linearized dimensionless momentum and heat transfer equations governing the perturbed state for variable-viscosity fluid obtained by Kalitzova-Kurteva et al. [29] are given by

$$f(z)\left[\left(D^2 - \alpha^2 + N^2 + 2ND\right)\left(D^2 - \alpha^2\right) + 2N^2\alpha^2\right]W = Pr^{-1}\omega\left(D^2 - \alpha^2\right)W, \tag{3}$$

$$\left[\omega - \left(D^2 - \alpha^2\right)\right]\Theta = -W. \tag{4}$$

The boundary conditions at the two boundaries comprise of,

$$W(0) = DW(0) = 0, \tag{5}$$

$$W(1) + \omega Z = 0, \tag{6}$$

$$f(1)\left[\left(D^2 - 3\alpha^2\right)DW(1) + N\left(D^2 + \alpha^2\right)W(1)\right] + \frac{\alpha^2(Bo + \alpha^2)Z}{Cr} = Pr^{-1}\omega DW(1), \tag{7}$$

$$f(1)\left(D^2 + \alpha^2\right)W(1) - \alpha^2 Ma\left[\theta(1) - Z\right] = 0, \tag{8}$$

$$D\Theta(1) + Bi\left[\Theta(1) - Z\right] = 0. \tag{9}$$

We set the boundary condition for the uniform temperature at the bottom boundary to include the controller rule and following Bau [13], we use a thermal feedback control mechanism to modify the heated surface temperature in proportion to the deviation of the fluid's temperature from its conductive value. The new boundary condition with the controller gain K is

$$\Theta(0) + K\Theta(1) = 0. \tag{10}$$

The operator $D = \mathrm{d}/\mathrm{d}z$ denotes differentiation with respect to the vertical coordinate z. $W = W(z)$, $\Theta = \Theta(z)$ and Z are the vertical variation of the velocity, temperature and the magnitude of the free surface deflection of the linear perturbations to the basic state, respectively, with α is the total wave number and ω is the complex growth rate. The function $f(z) = \mu/\mu_0$ is related to the viscosity variation N [29],

$$f(z) = \exp\left[N\left(z - 1 + \frac{T_0 - T_s}{\beta d}\right)\right], \tag{11}$$

where d is the depth of the liquid layer and T_s is the temperature at the free surface, where $T_s = T_w - \beta d$ is the temperature of the undisturbed state. If the reference temperature is the temperature at the free surface, $T_0 = T_s$ and $f(z) = \exp(N(z - 1))$.

The dimensionless parameters appearing in the problem are the Marangoni number $Ma = \epsilon\beta d^2/(\chi\mu_0)$, the Prandtl number $Pr = \mu_0/(\rho\chi)$, the Crispation number $Cr = \chi\mu_0/(\sigma_0 d)$, the Bond number $Bo = \rho g d^2/\sigma_0$, the Biot number $Bi = hd/\lambda$ and the viscosity parameter $N = \gamma\beta d$, where β is a positive constant, ρ is the density of the fluid, λ is the thermal conductivity, χ is the thermal diffusivity, h is the heat transfer coefficient between the liquid and gas phases and g is the gravitational acceleration.

3. Solution to linearized equations

The system (3)–(10) is solved analytically using the symbolic algebra package MAPLE to find Ma that determines the critical value for the onset of convection. The solutions for the amplitudes $W(z)$ and $\Theta(z)$ are in the form of hypergeometric functions in powers of $\omega\exp(-Nz)/(NPr)$ with series representations [22]

$$W(z) = \sum_{j=1}^{4} A_j \exp(a_j z) \sum_{n=0}^{\infty} B_{n,j} \exp(-nNz), \tag{12}$$

Table 1
The difference $|S_k - S_j|$, $j < k$, for $\overline{Ma}$ when $K = 11$, $N = 0.1$, $\overline{Cr} = 0.001$, $\alpha = 0.5$, $Pr = 10$, $Bo = 0.1$, $Bi = 0.1$, and $\omega = 0.5i$.

$\lvert S_k - S_j\rvert$	Numerical difference
$\lvert S_{10} - S_5\rvert$	$0.298461 \times 10^{-4} + 0.287096i \times 10^{-3}$
$\lvert S_{15} - S_{10}\rvert$	$0.510005 \times 10^{-5} + 0.236075i \times 10^{-4}$
$\lvert S_{20} - S_{15}\rvert$	$0.386114 \times 10^{-8} + 0.966914i \times 10^{-9}$
$\lvert S_{25} - S_{20}\rvert$	$0.384639 \times 10^{-16} + 0.139882i \times 10^{-15}$
$\lvert S_{30} - S_{25}\rvert$	$0.8212 \times 10^{-25} + 0.170212i \times 10^{-25}$
$\lvert S_{30} - S_{29}\rvert$	$0.1398 \times 10^{-25} + 0.617204i \times 10^{-25}$

$$\Theta(z) = \sum_{j=1}^{4} A_j \exp(a_j z) \sum_{n=0}^{\infty} C_{n,j} \exp(-nNz) + A_5 \exp\left(\sqrt{\omega + \alpha^2} z\right) + A_6 \exp\left(-\sqrt{\omega + \alpha^2} z\right), \tag{13}$$

where

$$a_1 = -\frac{N}{2} + k_1 + ik_2, \qquad a_2 = -\frac{N}{2} + k_1 - ik_2,$$
$$a_3 = -\frac{N}{2} - k_1 + ik_2, \qquad a_4 = -\frac{N}{2} - k_1 - ik_2, \tag{14}$$

with

$$k_1 = \frac{1}{4}\left(2k + 2N^2 + 8\alpha^2\right)^{1/2}, \tag{15}$$

$$k_2 = \frac{1}{4}\left(2k - 2N^2 - 8\alpha^2\right)^{1/2}, \tag{16}$$

$$k = \left(N^4 + 24\alpha^2 N^2 + 16\alpha^4\right)^{1/2}. \tag{17}$$

The coefficients of the series (12) and (13) are

$$B_{0,j} = 1, \tag{18}$$

$$B_{n+1,j} = \frac{1}{(n+1)!}\left(\frac{\omega}{NPr}\right)^{n+1} \frac{Q_{1,j} Q_{2,j} \cdots Q_{n+1,j}}{R_{1,j} R_{2,j} \cdots R_{n+1,j}}, \tag{19}$$

$$C_{n,j} = \frac{B_{n,j}}{(a_j - nN)^2 - (\omega + \alpha^2)}, \tag{20}$$

with

$$Q_{n+1,j} = (a_j - nN)^2 - \alpha^2, \tag{21}$$

$$R_{n,j} = -4a_j^3 + 6a_j^2(n-1)N + 2a_j\left[2\alpha^2 - (n-1)(2n-1)N^2\right] + \left[n(n-1)N^2 - 2\alpha^2\right](n-1)N. \tag{22}$$

We computed the solution from the series (12) and (13) using $n = 30$ with the MAPLE variable `Digits` controlling the number of significant digits set to 32 (see [22]). The coefficients A_j ($j = 1, 2, \ldots, 6$) and the expressions for Z and Ma are searched subject to the boundary conditions (5)–(10). The coefficient A_1 is found to be arbitrary. The series converge quickly as n increases, for example, Table 1 shows the numerical values $|S_k - S_j|$, $j < k$, for $\overline{Ma}$ where S_k denotes the summation of the series up to $n = k$ when $K = 11$, $N = 0.1$, $\overline{Cr} = 0.001$, $a = 0.5$, $Pr = 10$, $Bo = 0.1$, $Bi = 0.1$, and $\omega = 0.5i$.

The analytical solution above is based on the reference viscosity at the free surface $\mu_0 = \mu_s$. Since a variable-viscosity fluid is more stable than fluid with viscosity μ_w (viscosity at the wall) and less stable than a fluid with viscosity μ_s (viscosity at the free surface) [22,28], we use the mean value of the viscosities at both boundaries $\overline{\mu} = (\mu_s + \mu_w)/2$. Therefore, the mean Marangoni number $\overline{Ma}$ and the corresponding mean Crispation number $\overline{Cr}$ can be determined by the relations [22],

$$\overline{Ma} = \frac{2Ma}{1 + \exp(-N)}, \tag{23}$$

$$\overline{Cr} = \frac{Cr[1 + \exp(-N)]}{2}, \tag{24}$$

and the marginal stability curves for both steady and oscillatory convection are plotted from $\overline{Ma} = \overline{Ma}(\alpha, \omega, N, Bi, Bo, \overline{Cr}, Pr)$. The effects of controller gain and viscosity variation on the onset of convection are assessed based on the marginal stability curves. The critical parameter values for the situations when steady and oscillatory modes coexist and the bifurcation to oscillatory from steady for nondeformable surface will be determined.

4. Results and discussion

In this section, we present the results graphically to show the effects of the controller gain K, the viscosity parameter N, the Prandtl number Pr and the Crispation number Cr on the steady and oscillatory marginal curves for nondeformable surface, $Cr = 0$, and deformable surface $Cr \neq 0$. We consider the effects of the parameters on the time-dependent instability in the case of constant temperature at the rigid plate or the so-called conducting case. The stability state is determined by the Marangoni number $\overline{Ma} = \overline{Ma}_r + i\overline{Ma}_i$ and the growth rate $\omega = \omega_r + i\omega_i$, where $\omega_r = \mathrm{Re}(\omega)$ and $\omega_i = \mathrm{Im}(\omega)$. ω_r is set to zero for the stability to be at the marginal state in which disturbances are neither amplified nor damped, and the convection sets in as stationary motion if $\omega_i = 0$ or as oscillatory motion if $\omega_i \neq 0$. We consider the physical parameters values in the ranges: $0 \leqslant \overline{Cr} \leqslant 0.01$, $0 < N \leqslant 8$, $10 \leqslant Pr \leqslant 1000$, $0 \leqslant Bi \leqslant 0.1$ and $0 \leqslant Bo \leqslant 0.1$. Note that $N = 8$ corresponds to fluids with large viscosity variation such as glycerol [22,27].

Slavtchev et al. [22] solved the special case of uncontrolled system ($K = 0$) and constant temperature at the wall for the steady and oscillatory convection. They only found stationary instability. The nontrivial solution for the time-dependent instability has negative Marangoni number which means that the oscillatory instability cannot appear if the heating is from below [13,21,22]. Setting $K = 0$, $Cr = 0$ and $N = 0$, the system (3)–(10) reduces to the Marangoni problem of Pearson [3] and one recovers the Marangoni problem of Takashima [21,20] when $K = 0$ and $N = 0$. Bau [13] studied the effect of a feedback control on Marangoni convection of [3] for constant viscosity $N = 0$ and nondeformable surface $Cr = 0$, and found that in the case of oscillatory instability, large controller gains have a destabilizing effect.

4.1. Nondeformable surface

For a nondeformable surface and a constant temperature at the rigid wall with a feedback controller, the characteristics of the steady and oscillatory instabilities are illustrated in Figs. 1–5.

Fig. 1 shows the locus of (ω_r, ω_i) curves that satisfy the condition $\overline{Ma}_i = 0$. If $\omega_r < 0$ oscillatory modes decay in time and if $\omega_r > 0$ the oscillatory disturbances grow. Each point on the curve is associated with a Marangoni number $\overline{Ma} = \overline{Ma}_r$ of either positive, negative or zero value. Some values of $\overline{Ma}$ are indicated by circles at several points on the curves. As K increases, the curve moves upwards and evidently, increasing the controller gain K induces oscillatory convection. The curve for $\alpha = 0.1$, $K = 10$ is above the curve for $\alpha = 1.0$, $K = 10$ and therefore long wavelengths are less stable than shorter wavelengths.

Fig. 2 depicts the steady and oscillatory curves for $N = 1$ for several values of K. The critical Marangoni numbers $\overline{Ma}_c$ are marked by solid circles which determine the global minima of the steady and oscillatory curves. The steady curve for $K = 11.6$ is not plotted since it does not differ visibly from the steady curve for

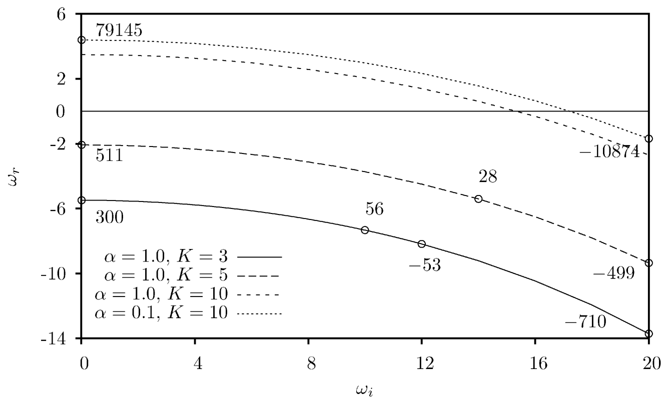

Fig. 1. Locus of (ω_i, ω_r) curves for several values of K and α when $\overline{Ma}_i = 0$, $Pr = 10$, $N = 1$ and $Bi = Bo = Cr = 0$.

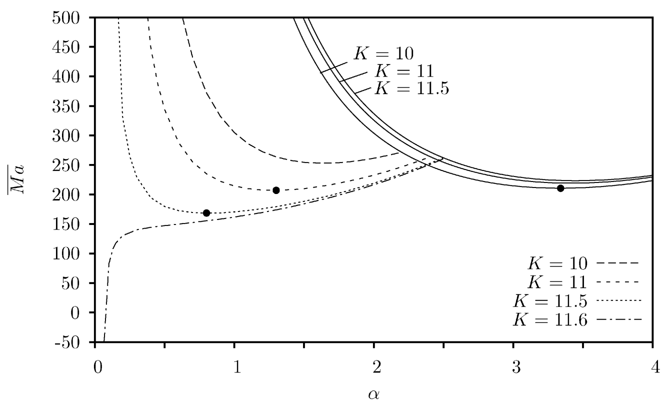

Fig. 2. Marginal curves for steady (solid lines) and oscillatory (dashed lines) for several K when $N = 1$, $Bi = \overline{Cr} = Bo = 0$ and $Pr = 10$. Global minima (critical Marangoni numbers $\overline{Ma}_c$) are marked by solid circles.

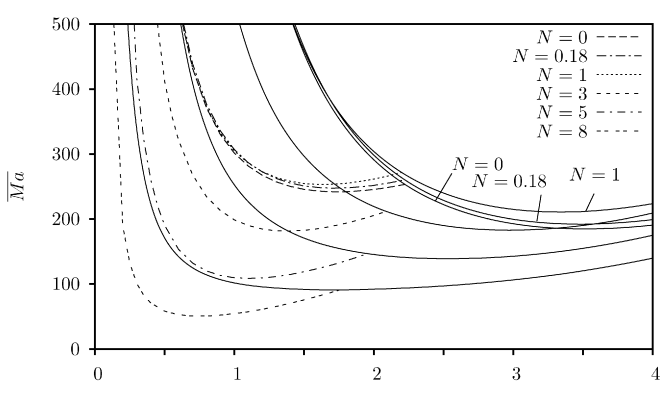

Fig. 3. Steady (solid lines) and oscillatory (dashed lines) marginal curves for several N when $Pr = 10$, $Bi = \overline{Cr} = Bo = 0$ and $K = 10$.

$K = 11.5$. In the case of oscillatory curves for nondeformable surface, we observed similar oscillatory behavior for variable viscosity in comparison to constant viscosity, cf. Bau [13]. We note that $\overline{Ma}_c$ is on the steady marginal curve for $K = 10$ and on the oscillatory curves for $K = 11$ and $K = 11.5$. When $K = 11.6$ there is no critical Marangoni number and the system is unstable at all positive Marangoni numbers.

As shown in Fig. 3, for $N = 3$, $N = 5$ and $N = 8$ with the other parameters values fixed, oscillatory instabilities can occur. The critical Marangoni number for the case $N = 1$ is higher than the critical values for $N = 0.18$ and $N = 0$. It shows that constant viscosity fluids are less stable than fluids with small viscosity variation. Thus, small viscosity variation has a stabilizing effect but further increment of the viscosity parameter N has a destabilizing effect. For a fixed value of K, there exists a critical value N_c to mark the coexistence of steady and oscillatory modes. A fixed value of controller gain definitely cannot maintain the stationary stability for all viscosity groups.

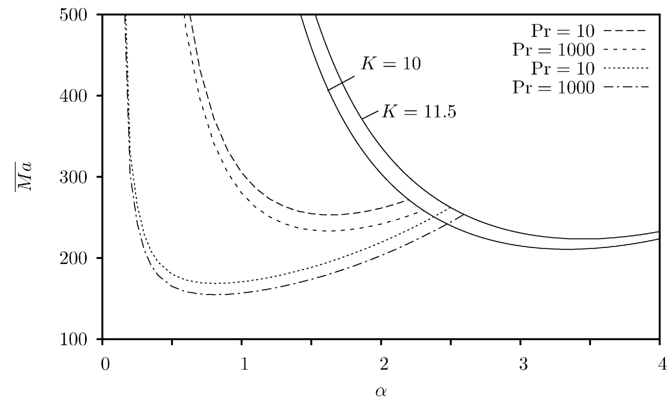

Fig. 4. Steady and oscillatory marginal curves for $Pr = 10, 1000$ when $N = 1$, $Bi = \overline{Cr} = Bo = 0$ and $K = 10, 11.5$.

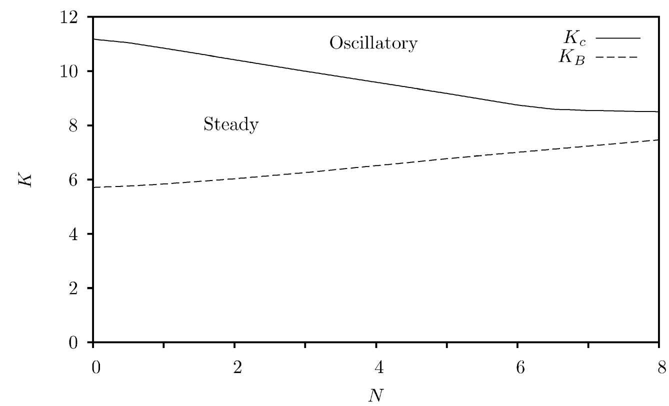

Fig. 5. Critical values of K_B when bifurcation from steady to oscillatory convection starts and K_c when both steady and oscillatory modes occur for the case $Bi = \overline{Cr} = Bo = 0$ and $Pr = 10$.

In Fig. 4 we observe that increasing Pr or K shifts the oscillatory curves downwards. When $K = 10$, the steady convection is preferred and when $K = 11.5$ oscillatory convection dominates.

As K increases, steady convection is delayed but eventually when the value of the controller gain is large enough, bifurcation to oscillatory instability starts to appear. The line K_B in Fig. 5 shows the critical value of K when the oscillatory curve starts to bifurcate from the stationary curve. Bifurcation to oscillatory mode starts at a larger K for large N. For a fixed N, a further increment in K will shift the global minimum on the steady curve to the oscillatory curve. Hence, there is another critical value, we denote as K_c, at which the oscillatory and steady instabilities occur simultaneously. The line K_c in Fig. 5 separates the regions in which stationary or oscillatory convection is preferred. When $K < K_c$ convection is steady for all N and when $K > K_c$ convection is oscillatory. K_c decreases as N increases, hence, oscillatory instability dominates at a smaller K for large N. Therefore, in the cases investigated, one has to choose $K < K_c$ to prevent the occurrence of oscillatory instability and to stabilize the system.

4.2. Deformable surface

For the uncontrolled system of Marangoni problem for the conducting case considered by Bau [13] with constant viscosity and

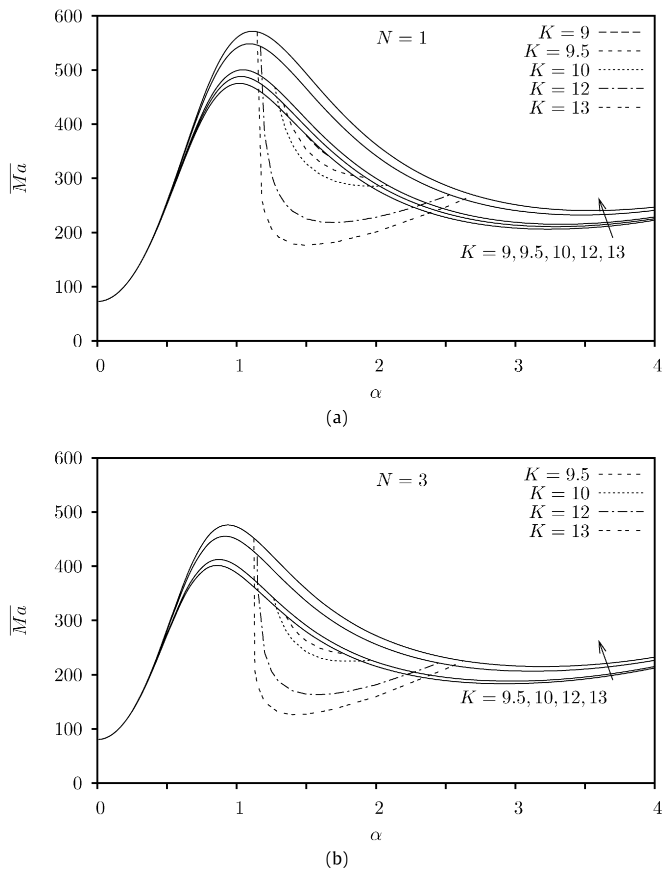

Fig. 6. Steady (solid lines) and oscillatory (dashed lines) marginal curves (a) $N = 1$ and (b) $N = 3$ for various K when $Pr = 10$, $Bi = Bo = 0.1$ and $\overline{Cr} = 0.001$.

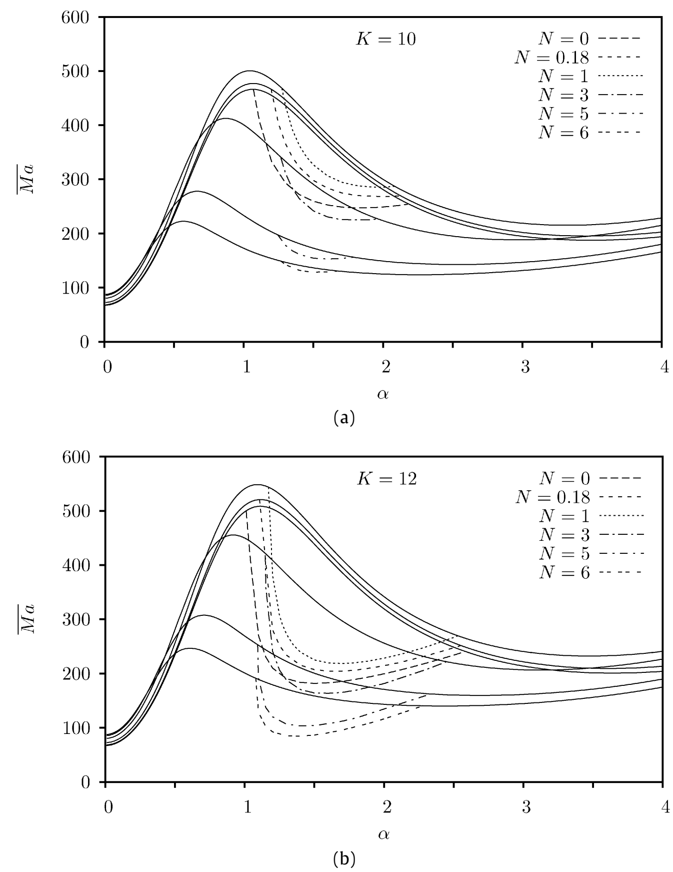

Fig. 7. Steady (solid lines) and oscillatory (dashed lines) marginal curves (a) $K = 10$ and (b) $K = 12$ for various N when $Pr = 10$, $Bi = Bo = 0.1$ and $\overline{Cr} = 0.001$.

Slavtchev et al. [22] for variable viscosity, the oscillatory instabilities cannot appear. Bau [13] only considered the case of flat free surface for oscillatory convection and demonstrated that large controller gains promote oscillatory instabilities. In this section, we present the results for the controlled system (3)–(10) for the case of deformable surface.

Fig. 6 shows the steady and oscillatory marginal curves for various K when $N = 1$ and $N = 3$. The global minima are at $\alpha = 0$. However, in practical situations in which the fluid layer is confined in a finite size container, the admissible wave numbers are nonzero. Thus, when one considers the critical values in the shorter wavelength regions ($\alpha > 0$), it can be observed that for small values of K, only steady convection appears but bifurcations to oscillatory convection occurs for relatively large K, for examples $K = 12$ and $K = 13$ for both $N = 1$ and $N = 3$. Large controller gains have a destabilizing effect with respect to oscillatory instabilities.

As depicted in Fig. 7, for the cases of viscosity group considered, when $K = 10$ only stationary convection sets in and oscillatory curves disappear as N increases. Similarly, as observed in the case of nondeformable surface, the minimum of the oscillatory curve for $N = 0$ is less than the minimum for $N = 0.18$ and $N = 1$. Small viscosity variation stabilizes the liquid layer at shorter wavelengths. When $K = 12$, oscillatory convection dominates and for $N = 6$, the minimum at the shorter wavelength is less than the minimum at the long wavelength. Therefore, for large K and large N, the oscillatory instability favors shorter wavelengths.

Figs. 8 and 9 illustrate the characteristics of the oscillatory marginal profiles for variations of Pr and $\overline{Cr}$, respectively. In Fig. 8, large Pr and large K inhibit oscillatory convective motions. As shown in Fig. 9, the effect of increasing $\overline{Cr}$ is to increase the minima of the oscillatory marginal curves but to decrease the global minima of the steady curves. At shorter wavelengths, low Crispation number induces oscillatory instability. Hence, large controller gains and low Crispation number have destabilizing effects on oscillatory instabilities.

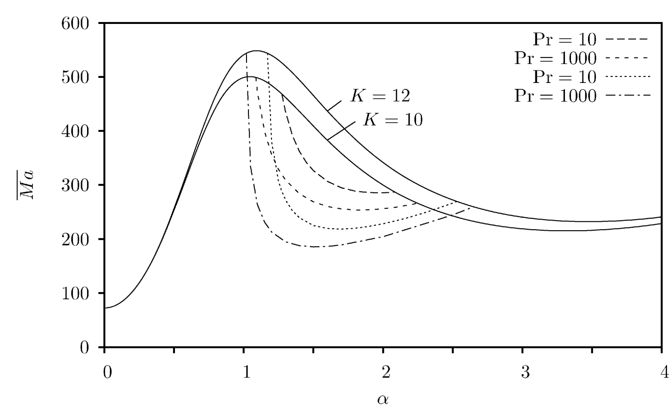

Fig. 8. Steady (solid line) and oscillatory (dashed lines) marginal curves for $Pr = 10, 1000$ when $N = 1$, $Bi = Bo = 0.1$, $\overline{Cr} = 0.001$ and $K = 10, 12$.

5. Conclusions

The effect of a feedback control strategy on the onset of oscillatory convection in an infinite horizontal layer of fluid with temperature-dependent viscosity has been investigated theoretically using linear stability analysis. In the case of a nondeformable surface, large controller gain and large viscosity variation induce oscillatory instability. In the case of a deformable surface, the os-

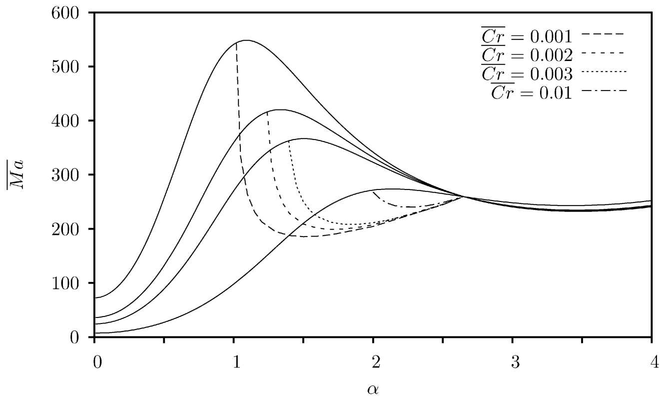

Fig. 9. Steady (solid line) and oscillatory (dashed lines) marginal curves for various $\overline{Cr}$ when $N = 1$, $Pr = 10$, $Bi = Bo = 0.1$ and $K = 12$.

cillatory convection sets in liquids with high surface tension (weak surface deformation), large viscosity variation, and large controller gains. Prandtl number also plays a role in destabilizing the liquid layer. Small controller gains of the thermal feedback control are effective in controlling the onset of steady and oscillatory instabilities, while large controller gains strongly influence the thermophysical properties of the liquid layer which significantly alter the convective flows and induce oscillatory instabilities.

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